The Paradigm of a transverse Area Structure for Lightfront Degrees of Freedom

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Abstract

It is shown that the degrees of freedom on the lightfront align themselves in such a way that it is enough to know their properties in one strip in lightlike direction of unit transversal size; a transversal tensor product foliation accounts for a Bekenstein-like area law of additive quantities as the entropy. This behavior is independent of details of models and we propose it as the basic local quantum physics analog of the Bekenstein area law. The field-coordinatization independent algebraic formulation in terms of operator algebras is essential for understanding this rearrangement of degrees of freedom in the process of (algebraic) lightfront holography.

1 Introduction

Peculiarities of lightfront and "$p \to \infty$ frame" behavior in particle physics have been noticed in many publications starting from the beginnings of the 70s [1]. In recent times we have seen a renewed interest in the subject as a result of the idea of holography [2] i.e. the conjecture that for certain geometric constellations it may be possible to encode degrees of freedoms and most of the properties of a QFT in d-spacetime dimensions into a suitably chosen lower dimensional geometric carrier. Although the first intuitive picture about such encoding came from consistency observations on rotational symmetric black holes, there were also arguments that a similar holographic encoding may occur in plane Minkowski space lightfront physics [3]; in fact such an idea receives additional support from the analogy of the Hawking effect caused by black holes with bifurcated horizons with the Unruh effect associated with the Rindler wedge in Minkowski spacetime [4] in the sense that if there exists an analogy in the appearance of a Hawking temperature, there should also be an analogous behavior of other thermal aspects as e.g. the entropy. Since a thermal interpretation imposed on the classical black hole situation requires the Bekenstein area law for entropy, and since the finite surface of a black hole corresponds to the infinite surface defined by the edge of the wedge, one would expect a constant surface density of the entropy of the half-lightfront (which should be identical to an appropriately defined wedge entropy, since it is easily shown that the wedge algebra is identical to its upper horizon algebra [5]).

The main aim of this note is to present new concepts and mathematical tools which show that this is not only an analogy between special situations in curved spacetime and some new aspects of lightfront physics (as compared to the way the lightfront and the $p \to \infty$ frame method were previously used in particle physics), but rather the start of a paradigmatic change in looking at local quantum physics[1]. In order to appreciate this statement the reader is reminded that for several decades there exist two ways of dealing with QFT which are mainly different in their interpretation, mathematical implementation of concepts and underlying philosophy but were based on a shared stock of principles. The standard approach (i.e. that of most textbooks and the vast majority of researchers) attributes a direct physical reality to pointlike quantum (suitably averaged in the sense of Bohr and Rosenfeld) fields, whereas the operator-algebraic setting is based on the idea that joint properties of algebras of operators which share the same spacetime localization is sufficient for extracting all the physics (analogous to the fundamental role of localization of events in counters without knowing the detailed inner working of an apparatus) [10].

The historically first observation which lend some respectability to the algebraic viewpoint is the natural explanation of the insensitivity of the S-matrix against local changes of the field-coordinatization. The support from the S-matrix viewpoint was strengthened by the recent observation that unitary crossing symmetric (this being the on-shell substitute for the missing off-shell Einstein causality) have maximally one system of local algebras if they have one at all [11]. However none of these observations really require the use of the algebraic viewpoint as much as the present one since e.g. the S-matrix continues to be expressible by LSZ formulas in terms of pointlike fields.

Although we are aware that the general level of knowledge about modular theory on which this paper is based is lagging far behind its importance in local quantum physics, we will make no pedagogical attempt in this short note to explain its mathematical content and physical achievements and even resist all temptations to comment on its fascinating history. For this the reader may consult preceding articles by the author or better read relevant sections in [10] and [12].

The present lightfront degree of freedom properties combine all these different aspects of particle physics, curved spacetime properties [4], basic quantum physics [14] and measurement aspects as well as philosophy of sciences [13].

[1]There are indications for an ongoing paradigmatic change in various other publications as [6][7][8][9].

2 The setting of local quantum physics and its adaptation to the lightfront

In the derivation of the time-dependent scattering theory and the analytic properties carried out in the 50s and 60s it became clear that pointlike fields are analogous to coordinates in differential geometry; the attribution of an physical reality to individual fields is a bit of an illusion which in certain cases may conceal basic intrinsic properties. This is in particular the case for QFT on the lightfront.

It is assumed that the reader knows some basic facts about the algebraic approach which describes QFT in terms of a collection (net) of spacetime indexed operator algebras [10] in a common Hilbert space fulfilling causality- covariance- and spectral- properties. These properties are adaptations of those which already appeared in the first non-Lagrangian setting of Wightman [15] and which even nowadays are often referred to as the linear Wightman requirements. Besides the linear Einstein causality (local commutativity) there are also causality requirements which are an algebraic substitute for an equation of motion namely the causal shadow property or primitive causality [16]

$$\mathcal{A}(\mathcal{O}) = \mathcal{A}(\mathcal{O}'') \tag{1}$$

in words the operator algebra (always weakly closed) localized in a simply connected region $\mathcal{O}$ equals the operator algebra of its causal completion (the spacelike disjoint of its spacelike disjoint $\mathcal{O}'$). Such properties have no natural expressions in terms of linear properties of field-correlations i.e. they are outside the linear Wightman properties which have their natural formulation in the operator algebra setting [16]. Since we will also be concerned with lower dimensional subalgebras as the lightfront algebra $\mathcal{A}(LF)$ we will also assume the *characteristic extension of causal shadow properties* of which the following relation is the most important case

$$\mathcal{A}(W) = \mathcal{A}(LF_+) \tag{2}$$

Classically its content and validity should be obvious: the field data in the (Rindler) wedge[2] $W = \left\{ x | \left| x^0 \right| < x^1, x^1 > 0 \right\}$ are determined in terms of the characteristic data on its upper causal horizon LF_+ (half the light front). W is the causal shadow of LF_+ since every particle or lightray which has passed through LF_+ must have before passed through W. But note that a region on LF which is smaller than the halfplane LF_+ does not cast any causal shadow at all; this is one of the peculiarities of the lightfront.

In local quantum physics the relation (2) would be meaningless without a proper definition of both sides. The operator algebra on the left hand side is defined in terms of the net-setting of AQFT as all operator algebras associated

[2] Our reference wedge will be always the $x^0 - x^1$-wedge, the wedge in any other position is obtained by applying Poincaré transformations.

with noncompact localization regions namely

$$\mathcal{A}(W) = \overline{\bigcup_{\mathcal{O}\subset W} A(\mathcal{O})} \tag{3}$$

where the closure is in the weak operator topology. Such algebras are required to obey the geometric Bisognano-Wichmann property $\mathcal{A}(W') = \mathcal{A}(W)'$ where W' is the geometric opposite (spacelike disjoint) of W and the dash on the operator algebra denotes as usual the von Neumann commutant algebra [10]. The right hand side upper horizon $\mathcal{A}(LF_+)$ algebra is best defined in terms of the result of the following theorem on modular inclusion

Theorem 1 *(Wiesbrock, [17][12]) Let $W_{e_+} \subset W$ be the lightlike translated wedge algebra $e_+ = (1,1,0,0)$. The inclusion of operator algebras*

$$\mathcal{A}(W_{e_+}) \equiv AdU(e_+)\mathcal{A}(W) \subset \mathcal{A}(W) \tag{4}$$

is "modular" i.e. the modular unitary Δ_W^{it} of the standard pair $(\mathcal{A}(W_{e_+}),\Omega)$ "compresses" the smaller algebra

$$\sigma_{t,W}(\mathcal{A}(W_{e_+})) \equiv Ad\Delta_W^{it}\mathcal{A}(W_{e_+}) \subset \mathcal{A}(W_{e_+}),\ t>0 \tag{5}$$

In this case the original positive energy lightray translation $U_{e_+}(a) \equiv U(ae_+)$ can be recovered from the two modular unitary groups $\Delta_W^{it}, \Delta_{W_{e_+}}^{it}$, and the lightlike translation $U_{e_+}(a)$ together with the modular group Δ_W^{it}. obey the Borchers translation-dilation commutation relation

$$\Delta_W^{it}U_{e_+}(a) = U_{e_+}(e^{-2\pi t}a)\Delta_W^{it}$$

If the relative commutant

$$\mathcal{A}(W_{e_+})' \cap \mathcal{A}(W) \tag{6}$$

is also standard with respect to the vacuum Ω (in which case the modular inclusion is called "standard"), then

$$\begin{aligned} \mathcal{A}(LF(0,1)) &\equiv \mathcal{A}(W_{e_+})' \cap \mathcal{A}(W) \\ \mathcal{A}(LF_+) &\equiv \overline{\bigcup_{a>0} AdU_{e_+}(a)\mathcal{A}(LF(0,1))} \\ \mathcal{A}(LF) &= \mathcal{A}(LF_+) \vee AdJ\mathcal{A}(LF_+) = \\ &= \mathcal{A}(W) \vee \mathcal{A}(W)' = \mathcal{A} \end{aligned} \tag{7}$$

Here J is the modular involution of the pair $(\mathcal{A}(W),\Omega)$. The resulting net structure on the lightfront algebra $\mathcal{A}(LF)$ is that of a generalized chiral theory; in addition to the lightray translation and the Borchers-associated dilation, the vacuum is invariant under a positive generator L_0 rotation. Its action on the original net is "fuzzy" (not representable by a diffeomorphism).

This theorem requires several comments. In its present form it is the adaptation of a completely abstract mathematical theorem on modular inclusion. In fact it comprises three closely intertwined theorems, the related Borchers- and Wiesbrock- theorems [12] and a theorem on the equivalence of standard modular inclusions with generalized chiral QFTs [18]. All the requirements concerning thy cyclic action of relative commutants (the standardness of modular inclusions) and the previous characteristic extension of the causal shadow property have been checked for free fields [19][20]; they are valid for all free field nets except the d=1+1 massless case for which one needs two kinds of chiral data. There is also a rich family of d=1+1 massive interacting theories for which all properties were checked[3]. It is a common practice to require the established modular properties for algebras of free fields also for interacting theories. Here we follow the common practice in AQFT to require such modular properties for all physically admissable models of particle physics.

The standard modular inclusion theorem resolves the problem of the longitudinal localization structure of the lightfront algebra, but it does nothing in the transverse direction of the edge of the bifurcated horizon. It is precisely the presence of these degrees of freedom which distinguish the generalized chiral theories of the previous theorem from those more familiar standard chiral theories which result from the chiral tensor decomposition of d=1+1 conformal theories. The former permit the nontrivial action (transverse translations and rotation) of automorphisms which, in lack of a transverse localization structure, are hard to distinguish from internal symmetries. However in the next section we will show how to construct a transversal localization using the methods of AQFT. The relevant Poincaré transformations are also those which one needs in order to reconstruct the original massive net from its holographic lightcone projection.

Let us collect those properties in form of a condition on which our analysis relies but which were not part of Wightman's framework [15] of what constitutes a QFT.

Condition 2 *Physically admissable models of local quantum physics fulfill in addition to the standard linear properties (local commutativity, covariance and spectral positivity) also the causal shadow property and its characteristic extension as well as the cyclicity of the relative commutants as needed in the previous modular inclusion theorem.*

What has been called standardness of $(\mathcal{A}(\mathcal{O}),\Omega)$ i.e. the cyclic generation of a dense subspace by applying $\mathcal{A}(\mathcal{O})$ to the vacuum and the fact that $\mathcal{A}(\mathcal{O})$ does not contain annihilators of the vacuum is a very general consequence of the algebraic setting known under the name of Reeh-Schlieder theorem. Its physical content is very surprising since it places QFT into a stark contrast to quantum mechanics. Whereas in quantum mechanics (in the multiplicative

[3] The check was however not on the level of operator algebras but only in the weaker form of formfactor spaces [23].

form of second quantization to facilitate the comparison with QFT) a quantization box divides the world into a tensor product between the inside and outside in such a way that the vacuum factorizes without entanglement, the QFT situation is the extreme opposite: from the algebra of an arbitrary small spacetime region (e.g. a small double cone as the prototype of a relativistic box) one can approximate any state from the vacuum. The denseness of what is cyclically generated from the vacuum is always guarantied as long as the localization regions has a nontrivial causal disjoint. In the physical literature this is often somewhat imprecisely referred to as the "operator-state(vector)" relation and in the mathematical theory of operator (von Neumann) algebras such an algebra is said to be in "general/standard position" with respect to the reference state vector. This situation is the starting point of the very rich mathematical Tomita-Takesaki modular theory of operator algebras. *All the significant differences to QM may be traced back to the phenomenon of vacuum polarization* which in turn is a direct consequence of the causality and spectral stability properties (positivity of timelike translation generators). A closely related refinement of the manifestation of vacuum polarization which is of direct relevance to particle physics is expressed in the following theorem on existence of properties of "polarization-free generators" (PFG) of localized one-particle creation operators. A PFG G is an (generally unbounded) operator affiliated with an operator algebra $\mathcal{A}(\mathcal{O})$ i.e. $G\eta\mathcal{A}(\mathcal{O})$ such that

$$G\Omega = 1 - particle\ vector$$

Theorem 3 *([21]) Wedge localized operator algebras in theories describing massive particles always possess affiliated PFGs $F\eta\mathcal{A}(W)$, whereas the existence of PFGs for algebras localized in causally closed subwedge regions imply the absence of interactions in those sectors which carry the charge of $F\Omega$; in fact their translates $F(x) = AdU(x)F$ are actually linearly related to free fields.*

This theorem is a strengthened form within the operator algebra setting of an old well-known theorem which characterizes free fields in terms of two-point functions [15] and which recently turned out to be relevant in connection with the protection mechanism in conformal SYM theories [22].

It is interesting for two reasons. On the one hand it gives an entirely intrinsic characterization of absence of interaction which does not depend on the chosen field coordinatization in the equivalence class of all local fields[4]. On the other hand the special role attributed to wedge algebras may be used as the start of a new constructive approach to QFT. Although the interaction is not directly visible on the level of PFG properties, it makes its appearance in the properties of the modular operators [23]. The TCP related modular involution J turns out to differ from its interaction-free form by the appearance of the scattering operator S_{scat} which thereby acquires a new role related to localization which completely escaped the old S-matrix philosophy (according to which on-shell

[4]Since the Wick-polynomials of free fields have quite complicated correlation functions, it would be difficult to assert the absence of interactions by just looking at correlations.

quantities do not reveal localization properties). Although it has up to now not been possible to show the existence of a QFT associated with a given admissable (unitary, crossing symmetric with the necessary analytic properties) S-matrix, the application of modular theory does lead to the uniqueness of the associated would be AQFT, i.e. there is either none or just one local quantum theory with an admissable S-matrix [11]. Knowing the structure of the wedge algebra one may construct the operator algebras of smaller regions by forming algebraic intersections (instead of restricting the support of smearing functions as in the standard approach). That this completely intrinsic algebraic way is practicable has been shown recently in some "well behaved" but nontrivial two-dimensional models of factorizable models. The mysterious nonlocal Zamolodchikov-Faddeev algebra which appeared as a computational tool acquires for the first time a spacetime interpretation: its Fourier transforms are the PFGs of the wedge algebra [23].

3 Tranverse Localization on the Lightfront and the return of Quantum Mechanics

We now turn to the issue of equipping the transverse lightfront directions with a localization structure. Our modular inclusion technique has created a generalized chiral net. The generalized chiral algebra associated with a longitudinal interval (a,b) should be pictured as the operator algebra of a d-2 dimensional strip of width $b-a$. We want to create a transversal net structure i.e. construct algebras $\mathcal{A}(\mathcal{O}_{LF})$ for compact $\mathcal{O}_{LF}\subset LF$ so that we can form two-sided open strips $S_T(-\infty,\infty)$ in the longitudinal direction with a finite transverse width $\Delta x_{\perp i}=T$ (here a geometric visualization for the case d=1+2 is helpful)

$$\mathcal{A}(\mathcal{S}_T(-\infty,\infty)) = \overline{\bigcup_{O_{LF}\subset\mathcal{S}_T(-\infty,\infty)} A(\mathcal{O}_{LF})} \tag{8}$$

The idea of introducing the horizontal $\mathcal{A}(\mathcal{O}_{LF})$ net structure consists in the application of Wigner little group transformations which preserves the direction of the horizontal lightray. In fact the idea was first used in a joint paper with Wiesbrock [24] in order to enrich the standard modular inclusion by L-tilted wedges as way to eventually recover the full net structure. These Wigner little group transformations place the transversely unresolved chiral holographic image of the wedge algebra into different relative positions and in this way become useful in the reconstruction of the original net structure from its holographic projection ("chiral scanning"). In particular the little-group "translations" (it is a Euclidean translation within the L-group) acts like a Galilei transformation in the lightfront plane

$$\mathbf{x}_\perp \rightarrow \mathbf{x}_\perp + \mathbf{v}x_+,\ \ x_+ \rightarrow x_+ \tag{9}$$

The d-2 dimensional strip in the direction of the transversal bifurcation edge is transformed into an inclined position within the lightfront and the intersections of the original with the transformed strip are the desired building blocks $\mathcal{O}_{LF}$ of the net of operator algebras on the lightfront from which the longitudinal strips (8) may be constructed. Following Driessler we now show that the foliation of the lightfront algebra $\mathcal{A}(LF)$ into longitudinal strips of unit d-2 volume T=1

$$\begin{aligned} \mathcal{A} = \mathcal{A}(LF) &= \overline{\bigcup_i \mathcal{A}(\mathcal{S}_i(-\infty,\infty))} \\ LF = \overline{\cup_i \mathcal{S}_i(-\infty,\infty)},\ &\mathcal{S}_i \cap \mathcal{S}_j = \emptyset,\, i \neq j \end{aligned} \tag{10}$$

has a tensor product structure.

Theorem 4 *([19]) The transversal foliation of the lightfront algebra is a tensor-product factorization*

$$\begin{aligned} A(LF) &= \bar{\otimes}_i \mathcal{A}(\mathcal{S}_i(-\infty,\infty)) \\ H &= \bar{\otimes}_i H_i,\ \Omega = \bar{\otimes}_i \Omega_i \\ H_i &= \overline{\mathcal{A}(\mathcal{S}_i(-\infty,\infty))\Omega} \end{aligned}$$

Proof. The proof is based on the existence of a transversal "get away" shift, i.e. by translating the strip transversely outside itself we obtain two commuting strip algebras. According to an old result of [26] one knows that each such algebra has a (lightlike) translation-invariant center and the lightlike translation acts as an inner automorphism. We use the projector onto the subspace which the strip algebra generates cyclically from the vacuum. Now look at the analytic properties associated with the following relation between an operator $A \in \mathcal{A}(strip)$ and another one from the commutant $A' \in \mathcal{A}(strip)'$

$$\left\langle 0 \left| A U_{e_+}(a) A' \right| 0 \right\rangle = \left\langle 0 \left| A' U^*_{e_+}(a) A \right| 0 \right\rangle \tag{11}$$

But using the analyticity due to the positivity of the generator we obtain via the Liouville theorem the constancy in a and hence

$$\left\langle 0 \left| A A' \right| 0 \right\rangle = \left\langle 0 \left| A \right| 0 \right\rangle \left\langle 0 \left| A' \right| 0 \right\rangle \tag{12}$$

i.e. the type I_∞ tensor factorization [19], as well as the stronger statement that the vacuum has no entanglement with respect to the inside-outside tensor factorization. Recursive application together with the fact that (7) $\mathcal{A}(LF) = \mathcal{A}$ yields the tensor factorization and absence of vacuum entanglement in the tiling of the lightfront by strips

$$\Omega = \overline{\bigotimes_i} \Omega_i \tag{13}$$

The one-sided strip algebras on LF_+ are easily shown to be type hyperfinite type III_1 algebras within the respective tensor factors. Since the argument is the same as that for wedges, it will not be repeated here [12]. ■

This theorem requires some comments. In local quantum physics one encounters several situations in which the notions of "causal disjoint" becomes synonymous with "nonoverlapping". For standard chiral theories on S^1 this is well-known, but it also holds for conformal observables living in the Dirac-Weyl compactified Minkowski spacetime if "nonoverlapping" is interpreted as "nonoverlapping of the lightlike prolongation" (Huygens principle). Via the CQFT-AdS isomorphism [6] this simplified causality picture is inherited by observables in anti deSitter spacetime. But none of these cases has such far-reaching factorization properties as the local quantum physics on the lightfront.

The usefulness of auxiliary constructs and associated (holographic) reprocessing of degrees of freedom depends very much on how the degrees of freedom "align" themselves after they received their new "spacetime-indexing". In this respect the holographic reprocessing onto causal horizons (for the rotational case see next section) is extraordinarily rich because it preempts the transversal factorization structure which is necessary for area behavior (unit transversal volume) of thermodynamic quantities (entropy, chemical potential...).

This extremely useful role of lightfront holography in the quest for a generic quantum Bekenstein area law does however not extend to recent speculations about the role of "branes" in local quantum physics. In that case the causal shadow extensions into the ambient spacetime prevent their interpretation as independent physical objects. This raises the question whether brane concepts together with the closely related Kaluza-Klein reduction and many other string-theory supported ideas can be consistent with the causal shadow property and the control of vacuum fluctuations for small additional spatial dimensions outside the quasiclassical approximation. For physicists who believe that paradoxa and contradictions contain the enigmatic force for progress, these are very good times.

It is worthwhile to point out that the use of algebraic lightfront description goes far beyond its connection with the Bekenstein area law. It constitutes a new tool of local quantum physics which offers all the advantages of the old equal time canonical formalism without suffering from its short distance limitations. Whereas the canonical formalism required the finiteness of the wave function renormalization constants Z (finiteness of the integral over the Kallen-Lehmann spectral functions) and therefore excludes all properly renormalizable models, the algebraic holography based on modular inclusion avoids the restriction of fields to space- or lightlike- subspaces (which is the cause of the short-distance restriction). Furthermore the chiral theories obtained by this construction are of an extreme kinematical and universal kind. This is because the modular inclusion method encodes the spacelike Boson/Fermi structure in the original formulation into (half)integer dilatation spectrum on the horizon. In fact the lightfront algebra can be described in terms of canonical (half)integer dimension generating fields [27], but they have no obvious connection to the pointlike fields which may have generated the original algebras; the latter have in general anomalous short distance dimension. The scale spectrum of the kinematical chiral theory results from the statistics and is not related to the anomalous short distance scale spectrum of the original theory. Although both the scaling short distance limit and the lightfront holographic limit exhibit conformal symmetry, the lightfront theory is much more "universal" than the short distance universality classes used in the description of critical phenomena. In addition the latter allow no (mathematically controllable) return to the original theory (they live in different Hilbert spaces) whereas the lightfront holography reveals its massive origin upon application of the Poincaré covariances. In short, the algebraic lightfront formalism seems to be the long looked for dynamical "dorado": a universal kinematical operator algebra which is reprocessed into the rich world of quantum field theoretic models by differently acting automorphisms. As a result of their short distance limitations neither the canonical formalism not the closely related Euclidean action method could play this role, although they both served (and still serve) as useful artistic catalyzers of thoughts about particle physics[5].

The presentation of the *transversal return to (vacuum-polarization-free) quantum mechanics* would be incomplete without indicating of what kind of entropy one assigns to the unit longitudinal half strip algebra on the horizon $\mathcal{A}(S(0,\infty)) \subset \mathcal{A}(LF_+)$. A direct definition is meaningless since these algebras are (as the result of the longitudinal vacuum polarization) hyperfinite type III_1 operator algebras which do not allow tracial states/weights. The aim would be to factorize the original vacuum into tensor product vacua corresponding to the bifurcated right/left horizons $\mathcal{A}(LF_\pm)$. But, as was shown in Wald's book ([4] and further references therein) and repeated in later publications these very nature of these operators algebras does not permit this unless one cuts off degrees of freedom which would be harming the very problem which one wants to solve. What one should do is to rearrange degrees of freedom (as it already was done in the holographic projection onto the lightfront) rather than throw away some of them. Precisely this is achieved by the splitting method which consists in creating a small bit finite distance a between them[6]. Here the analogy to the inside/outside nonrelativistic quantization box (say in the multiplicative formulation of 2^{nd} quantization in order to make the analogy closer) breaks down, because by removing some spatial localization region (for all times) one is really dumping degrees of freedom. A closer examination of the QFT situation reveals is that what splitting does is to avoid uncontrollable divergencies which are invariably created by vacuum fluctuations if they are forced to take place directly in a surface (or in a point in the chiral case). Both localizations regions in this way have a fuzzy extension into the "collar" and this fuzzy splitting is only a rearrangement and does not involve throwing away degrees of freedom.

[5] The correct renormalized answers only fulfill Einstein causality but are neither canonical nor Feynman-Kac representable.

[6] The splitting can also be formulated directly in the higher dimensional theory where it corresponds to putting a "collar" between a double cone ($\sim$ "relativistic box") and its causal complement [10].

In the chiral case there is an additional point which one must pay attention to. The creation of a small distance a between the two sides separated by the transversal edge is not enough, as a result of the unavoidable conformal compactification the two sides are still sticking together at lightlike infinity. A second small a-interval at infinity will do the job and fortunately the entropy would not change (at least in the limit $a \to 0$) if one completes the separation of the two sides at another place. For more details about this and the description of the entanglement entropy of the vacuum with respect to its split tensor product factorization in terms if a relative entropy we refer to [5]. Although the resulting formulas show the logarithmic divergence for $a \to 0$, their explicit evaluation or even good estimate of the localization entropy per unit transverse area has yet to be done.

This quantum divergence in the limit $a \to \infty$ pertains of course also to black hole horizons and this raises the interesting question how the classically assigned Bekenstein entropy area density is related to the present quantum entropy area density. Since the Bekenstein expression follows from the quantum Hawking temperature by requiring the validity of a certain form of suitably interpreted thermodynamic basic laws which involve several terms, it is natural to conjecture that there should be a quantum version with several terms in which the $a \to 0$ divergence should occur in the same way in each term so that it can be scaled away. This would be more a rescaling than a renormalization (which would involve the introduction of counter terms). The problem is important, but its solution has to be left to the future.

4 Comment on rotational horizons

Let us now consider the more difficult task of a rotationally symmetric double cone and its noncompact causal disjoint. One immediately notices some analogies to the previous planar bifurcated causal horizon. The characteristic causal shadow of the lower lightcone horizon $h_-(C)$ of the unit double cone C (placed symmetric around the origin) agrees with the double cone and its complement on the mantle of the future lightcone $h_+(V_+) = h(V_+).\backslash h_-(C)$ is the rotational analog of the upper horizon of the Rindler wedge. Instead of the planar longitudinal strips we now consider strips into radial directions with a fixed space angle opening which start at the rotationally symmetric edge of bifurcation and extends to lightlike infinity. In this way the horizon $h_+(V_+)$ may be partitioned into radial strips $\mathcal{S}_i$ which cast no causal shadow and whose associated algebras mutually commute. This is the prerequisite for a tensor product foliation

$$\begin{aligned} &\overline{\mathcal{A}(h_+(V_+))\Omega} = H \\ &H = \overline{\bigotimes}_i H_i, \;\; H_i = \overline{\mathcal{A}(\mathcal{S}_i)\Omega} \end{aligned} \tag{14}$$

The crucial question is whether there exists a substitute for a lightlike translation with positive generator whose analytic properties implies the transversal (in the rotational sense) factorization of the vacuum into entanglement-free strip vacua in analogy to Driessler's theorem in the previous section.

It is precisely at this point where the analogy becomes somewhat opaque. Part of the difficulty results from the fact that the rotational causal horizon does not come with a Killing vector field (analogous to event horizons in curved spacetime which are not associated with Killing symmetries), except in case the QFT is massless. In that case the Ad-action of the modular unitary and the Tomita involution J of the unit double cone corresponds to the following point transformations (the origin is in the center of the unit double cone [10])

$$Ad\Delta^{i\frac{s}{2\pi}}: x_{\pm}(s) = \frac{(\cosh s)\, x_{\pm} + (\sinh s)}{(\sinh s)\, x_{\pm} + (\cosh s)},\ x_{\pm} \equiv x^0 \pm |\vec{x}| \tag{15}$$
$$J: x^i \to -\frac{x^i}{x^2},\ x^0 \to \frac{x^0}{x^2},\quad -1 < x_{\pm} < 1$$

Although an explicit description of the corresponding fuzzy modular transformations in the massive case is presently not possible[7], the modular objects become geometric in the holographic projection in which the double cone is projected onto its lower lightcone mantle and the spacelike disjoint projects on the infinite complementary outside part of the mantle.

Remark 5 *The restriction of the double-cone localized free massive field to the lower horizon (i.e. to the mantle of that part of the forward lightcone whose causal shadow is the double cone) follows exactly the method of restricting free field operators to LF_+ [28]: the limit $x_- = x^0 - |\vec{x}| \to 0$ in the plane wave factor (with the origin at the lower apex of the double cone) is compensated by an $\ln x_-$-increase in the spatial rapidity χ; the creation/annihilation operators remain unaffected. The resulting free field restriction is that of a massless field (the mass only remains as a scale factor in the exponential), however the physical mass is re-activated by acting with Poincaré transformations in the lightcone restriction.*

It is believed that the holographic lightcone projection of all massive theories admits this geometric modular group action on its holographic projection

$$Ad\Delta^{i\frac{s}{2\pi}}: x_{+}(s) = \frac{(\cosh s)\, x_{+} + (\sinh s)}{(\sinh s)\, x_{+} + (\cosh s)} \tag{16}$$
$$x_- = 0,\ \ x_+ \equiv x^0 + |\vec{x}|\,,\ |x_+| \leq 1 \tag{17}$$

and on the holographic projection of its causal disjoint $x_- = 0,\ x_+ \geq 1$. This leads to the interesting conjecture that the theorem about modular inclusion of the previous section may have a partially geometric counterpart in which

[7] The conjecture that these transformation correspond to double cone (and its causal disjoint) support preserving test function transformations of the pseudo-differential kind [24] still stands.

two fuzzy modular groups[8] may nevertheless lead to a geometric Borchers pair (positive lightlike translation, dilation) on the lightcone horizon. Further work is necessary to clarify the situations of causal (event) horizons without Killing symmetries.

5 Miscellaneous

My use of the adjective “paradigmatic” in the title requires more justification.

In most uses of methods of AQFT there was always a way to see things (perhaps not in the most elegant fashion) in terms of field-coordinatizations. In the case at hand, as a result of severe short distance limitations which rule out to define holographic lightfront projections by restricting pointlike fields (except for free fields), this is not possible.

The older folks had learned how to live with this somewhat artistic situation between using something as a mental catalyzer for a computation (canonical formalism, functional integration) but being prevented by mathematical facts to make mathematical sense out of it in the presence of field theoretic short distance behavior. It is a typical humane reaction to suppress something which one cannot change anyhow. But in certain moments, notably if one had to teach a course on QFT right after a quantum mechanics course, one is always sadly reminded that one does not use a formalism where it would be legitimate (but too clumsy to solve quantum mechanical models) and rather starts to use it when it looses its legitimacy. The problem did not disappear for the last 40 years and there is no hope that it ever does. So young folks are led by their innovative mentors who (not wanting any unrest about points which they considered unimportant for their very nice differential geometric use of functional integrals) simply imposed a kind of particle physics “Fatwa” on this issue. My tests with young physicists have shown that their mentors have succeeded almost completely. This is a bit sad because all the original enigmatic power which should be expected on historical grounds in such a situation has been (at least temporarily) squandered. In these notes we have paid utmost attention to bypass these pitfalls.

In the present counting of degrees of freedom issue I do not think that there exists a field-coordinate dependent Lagrangian way (if a reader finds one I would be extremely interested to know). The derivations of area laws in special models, either in string theory [29] or by imposing a classical Virasoro structure [31] on horizons or by rewriting canonical classical GR variables into loop-like variables [32] are too special and restricted in order to serve as a quantum explanation of the general Bekenstein area law. The fact is that most of these investigations are more on the quantum mechanical (or even classical) side since they tend to overlook the dominating role of vacuum polarization. This does not necessarily

[8] It has been conjectured that the action of fuzzy modular actions on testfunctions should be described by a special type of pseudo-differential operators which leaves their given $\mathcal{O}$-support and its causal disjoint invariant [30] but unforunately no progress has been obtained on this point.

mean that they are in contradiction to the present explanation because vacuum polarization effects could be effectively transferred into geometrical aspects. One needs a good geometric situation (Killing vectors, bifurcated horizons) in order to have a "classical marking" of an otherwise hardly visible pure quantum effect, and models of curved spacetime could help to do just this.

In fact this links up very nicely with a recent stunning discovery about the true nature of QFT [9][8]. Whereas prior to that observation the curved spacetime aspect were always considered as part of the specification of the model, the new view of QFT is that of a functor between the categories of globally hyperbolic Lorentz signature manifolds (with the arrows being isometric embedding) and that of operator algebras (with the arrows being morphisms). In other words the curved spacetime aspect is not an additional property but belongs to the very definition of what constitutes a QFT. In order to illustrate this paradigmatic change of thinking about QFT let me compare the abstract form of QFT with a kind of unstructured mold of degrees of freedom, similar to stem-cells with the spacetime acting as agents for structuring the degree of freedoms by forcing it to align in a certain way. If anything, the present modular supported holography deepens this surprising new view in that it permits to change the spacetime indexing in an even more radical way by allowing lower dimensional sets.

This view de-emphasizes somewhat the importance of black holes as a direct entrance ticket into QG. But not completely, because there is the very interesting message of an apparent very *deep connection of thermal physics with geometry* at the place where one wants to see new quantum gravity degrees of freedom. This has already been foreshadowed by recent results on the construction of external and internal symmetries and spacetime geometry from the relative position of operator algebras and in particular the emergence of infinite dimensional fuzzy analogs of diffeomorphism groups (including the Poincaré and conformal diffeomorphisms) from modular inclusions[9] and intersections of algebras point into the same direction [17][33][7][34].

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